10.1 Conditions for interference
The term interference refers to any situation in which two or more waves overlap in space. The resultant wave at any point at any instant of time is governed by the principle of superposition. However, not all interference patterns are sustainable. For instance, the interference patterns formed by two light sources emitting waves randomly will keep on changing. To observe sustained interference pattern in light waves, the following conditions must be met:

1. The sources must be **coherent**; that is, they must maintain a constant phase with respect to each other.
2. The sources must be **monochromatic**; that is, of a single wavelength.

A common method to produce two coherent light sources is to use one monochromatic source to illuminate a screen containing two small openings (usually in the shape of slits). The light emerging from the two slits is coherent.

10.2 Young’s double-slit experiment
The first serious challenge to the particle theory of light was made by the English scientist Thomas Young in 1803. Young knew that if two sound waves of equal intensity, but 180° out of phase, reach the ear then they cancel one another out, and no sound is heard. Young reasoned that if light were a wave, then a similar interference effect should occur for light.

In Young's experiment, two parallel slits (separated by a distance \(d\)) serve as a pair of coherent light sources; the waves eventually hit a screen (distance \(L\) from the slits) to form an interference pattern. The experimental setup is sketched in the diagram below.

We use the approximation that \(r_1\) is parallel to \(r_2\) since \(L \gg d\). Thus, the path difference between the two rays is \(\Delta r = r_2 - r_1 = d \sin \theta\).

The light waves emanating from each slit are superposed on the screen. If the waves are 180° out of phase then destructive interference occurs, resulting in a dark patch/fringe/band on the screen. On the other hand, if the waves are completely in phase then constructive interference occurs, resulting in a light patch on the screen.

The point \(O\) on the screen, which lies exactly opposite to the center point of the two slits, as shown in the diagram, is obviously associated with a bright patch. This follows because the path-lengths from each slit to this point are the same and the emanating waves from each slit are initially in phase.

The general condition for constructive interference on the screen is simply that the path difference \(\Delta r\) between the two waves is an integer number of wavelengths. In other words,

\[
d \sin \theta = m \lambda \quad (m = 0, \pm 1, \pm 2, \ldots) \quad \text{(not just +1...)}
\]

\(m\) is called the order number, the \(m^{th}\) maximum.
Likewise, the general condition for destructive interference on the screen is that the difference in path-length between the two waves is a half-integer number of wavelengths. In other words, 
\[ d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \]  
(m = 0, ±1, ±2, …) 

Note: the 1st dark fringe is with m = 0, not m = 1.

If we consider points P where its distance y from the center point O is small compared to L, that is y << L or \( \theta \) is small, then we can use approximation \( \sin \theta \approx \tan \theta \).

Therefore, the positions of the bright fringes measured from O are given by 
\[ d \sin \theta = m \lambda \]

\[ d \left( \frac{y_{\text{max}}}{L} \right) = m \lambda \]

\[ y_{\text{max}} = m \frac{\lambda L}{d} \]

Similarly, the dark fringes are located at 
\[ y_{\text{min}} = \left( m + \frac{1}{2} \right) \frac{\lambda L}{d} \]

It is clear that the interference pattern on the screen consists of alternating light and dark bands, running parallel to the slits.

Example 1: A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe (m = 2) is 4.5 cm from the center line. (a) Determine the wavelength of the light.

\[ d \sin \theta = m \lambda \]

\[ d \left( \frac{y}{L} \right) = m \lambda \]

\[ \lambda = \frac{yd}{mL} \]

\[ = \frac{(4.5 \times 10^{-2} \text{ m})(0.030 \times 10^{-3} \text{ m})}{2(1.2 \text{ m})} \]

\[ = 5.6 \times 10^{-7} \text{ m} = 560 \text{ nm} \]

(b) Calculate the distance between adjacent bright fringes

\[ y_{m+1} - y_m = (m + 1) \frac{\lambda L}{d} - m \frac{\lambda L}{d} \]

\[ = \frac{\lambda L}{d} \] (which does not depend on m)

\[ = \frac{(5.6 \times 10^{-7} \text{ m})(1.2 \text{ m})}{0.030 \times 10^{-3} \text{ m}} \]

\[ = 2.2 \times 10^{-2} \text{ m} = 2.2 \text{ cm} \]

Exercise 1: Estimate the distance between the central bright region and the third dark fringe on a screen 5.00 m from two double slits 0.500 mm apart illuminated by 500-nm light. Ans: 1.25 cm

Exercise 2: In a Young’s double-slit experiment using 350 nm light, a thin piece of Plexiglas (n = 1.51) covers one of the slits. If the center point on the screen is a dark spot, what is the minimum thickness of the Plexiglas? [Hint: The difference in number of wavelengths between the two waves arriving at the spot is half wavelength.]  
Ans: 343 nm
Example 2: A light source emits visible light of two wavelengths: $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.5$ m and $d = 0.025$ mm. Find the separation between the third-order bright fringes.

The values of the third-order ($m = 3$) bright fringe positions corresponding to these two wavelengths are

$$y_3 = m\frac{\lambda L}{d} = 3 \frac{(430 \times 10^{-9} \text{ m})(1.5 \text{ m})}{0.025 \times 10^{-3} \text{ m}} = 7.74 \times 10^{-2} \text{ m}$$

$$y'_3 = m\frac{\lambda' L}{d} = 3 \frac{(510 \times 10^{-9} \text{ m})(1.5 \text{ m})}{0.025 \times 10^{-3} \text{ m}} = 9.18 \times 10^{-2} \text{ m}$$

Hence, the separation between the two fringes is

$$\Delta y = y'_3 - y_3 = 3 \frac{(|\lambda| - \lambda')L}{d} = 3 \frac{(510 \times 10^{-9} \text{ m} - 430 \times 10^{-9} \text{ m})(1.5 \text{ m})}{0.025 \times 10^{-3} \text{ m}} = 1.44 \times 10^{-2} \text{ m} = 1.4 \text{ cm}$$

### 10.3 Intensity distribution of the double-slit interference pattern

We have seen the interference pattern, the dark and bright fringes, produced by two coherent sources of light in Young’s double-slit experiment. We now calculate the intensity of light at all points in the pattern, as a function of $\theta$.

The waves leaving the slits are in phase and have the same angular frequency $\omega$. When the waves arrive at point $P$ on the screen, they might not be in phase, but have a constant phase difference $\phi$.

Assuming the waves have the same amplitude $E_o$, and the total electric field $E$ at point $P$ can be written as

$$E = E_1 + E_2 = E_o \sin \omega t + E_o \sin(\omega t + \phi) = 2E_o \cos \left(\frac{\phi}{2}\right) \sin \left(\omega t + \frac{\phi}{2}\right)$$

More useful in practice that most light-detecting instruments measured is the intensity $I$; the intensity of a wave is defined as the average rate of energy transported by the wave, per unit area, across a surface perpendicular to the direction of propagation. In section 34.3: Energy carried by electromagnetic waves, light intensity is proportional to the time average of the square of electric field:

$$I \propto (E^2)_{\text{avg}} = \left[2E_o \cos \left(\frac{\phi}{2}\right)\right]^2$$

A path difference $\Delta r$ of one $\lambda$ corresponds to a phase difference $\phi$ of $2\pi$. Therefore,

$$\phi = \frac{2\pi}{\lambda} \Delta r = \frac{2\pi}{\lambda} d \sin \theta$$

And, the previous equation can be written as

$$I = I_o \cos^2 \left(\frac{\pi d \sin \theta}{\lambda}\right)$$
For small \( \theta \), \( \sin \theta \approx \tan \theta = \frac{y}{L} \).

Then,

\[
I = I_o \cos^2\left(\frac{m \lambda y}{L} \right)
\]

The intensity is maximum (=\( I_o \)) when cosine has a value \( \pm 1 \), \( y = m \lambda L \), or \( \phi = 2m\pi \).

\[
\cos\left(\frac{m \lambda y}{L} \right) = \pm 1 \quad \Rightarrow \quad y = \frac{m \lambda L}{d} \quad m = 0, \pm 1, \pm 2, \ldots
\]

Example 3: Two radio antennas separated by 10 m radiate in phase with each other at frequency 48 MHz. The intensity at a distance of 600 m in positive \( x \)-direction (i.e.: \( \theta = 0^\circ \)) is \( I_o = 0.030 \) W/m\(^2\).

(a) What is the intensity in the direction of \( \theta = 5.0^\circ \)?

\[
\lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{48 \times 10^6 \text{ s}^{-1}} = 6.25 \text{ m}
\]

\[
I = I_o \cos^2\left(\frac{\phi}{2}\right) = I_o \cos^2\left(\frac{m \lambda y}{\lambda} \right)
\]

\[
= (0.030 \text{ W/m}^2) \cos^2\left[\frac{\pi (10 \text{ m}) \sin 5.0^\circ}{6.25 \text{ m}}\right] = 0.025 \text{ W/m}^2
\]

(b) In what directions is the intensity zero?

The intensity is zero when the cosine has zero value:

\[
\cos^2\left(\frac{m \lambda y}{\lambda} \right) = 0
\]

\[
\frac{m \lambda y}{\lambda} = \pm \frac{1}{2} \pi, \pm \frac{3}{2} \pi, \pm \frac{5}{2} \pi, \ldots
\]

\[
\sin \theta = \pm \frac{\lambda}{2d}, \pm \frac{3\lambda}{2d}, \pm \frac{5\lambda}{2d}, \ldots
\]

\[
= \pm 0.3125, \pm 0.9375, \quad \pm 1.5625 \text{(ignored since } |\sin \theta| \leq 1) \]

\( \theta = \pm 18^\circ \) and \( \pm 70^\circ \)

Exercise: In a particular location in a Young’s interference pattern, the intensity on the screen is 64.0% of the maximum. (a) What is the minimum phase difference, in radians, between the light sources that produce this result? (b) Determine the path difference for 632.8-nm light.

\( \text{Ans: } 1.29 \text{ rad; 130 nm} \)